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### Mass Systematics Involving Low-Lying Excited States\*

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With 3 Figures

#### Introduction

For the evaluation of the nuclear binding energy surface it is helpful to consider the excited states together with the ground states<sup>1, 2</sup>. The (positive) total nuclear binding energy for any state is  $B = B_{g.s.} - E_x$  where "g. s." refers to the ground state and  $E_x$  is the excitation energy.

For simplicity the mass excess  $\Delta$  can be used instead, since the total nuclear binding energy is

$$B(A, T_\zeta) = \frac{1}{2} A[\Delta(n^1) + \Delta(H^1)] + T_\zeta[\Delta(n^1) - \Delta(H^1)] - \Delta(A, T_\zeta).$$

Both brackets are constant; the first term depends linearly on the mass number  $A$  and the second is constant since only sequences of nuclides with constant  $\zeta$ -component of the isobaric spin,  $T_\zeta$ , are considered. Therefore a straight line in the mass excess  $\Delta(A)$  for  $T_\zeta = \text{const.}$  means a linear relation in the total binding energy  $B(A)$ .

The steep increase of  $\Delta$  in the light mass region can be compensated for by plotting the sum of the mass excess with a straight line,  $\Delta + pA$ , where again linear relations are conserved.

#### Results

Fig. 1 presents such a graph for self-conjugate even nuclides with  $p = 1.5$  MeV for convenient display.

The  $2^+$  and  $4^+$  levels of  $\text{Ne}^{20}$ ,  $\text{Mg}^{24}$ , and  $\text{Si}^{28}$  lie on almost straight trends, but the ground states do not. Weak indication has been given by MORITA and TAKESHITA<sup>3</sup> for a level in  $\text{Ne}^{20}$  at 0.65 MeV — it happens to be just in the right position to fit a trend — which has not been found in

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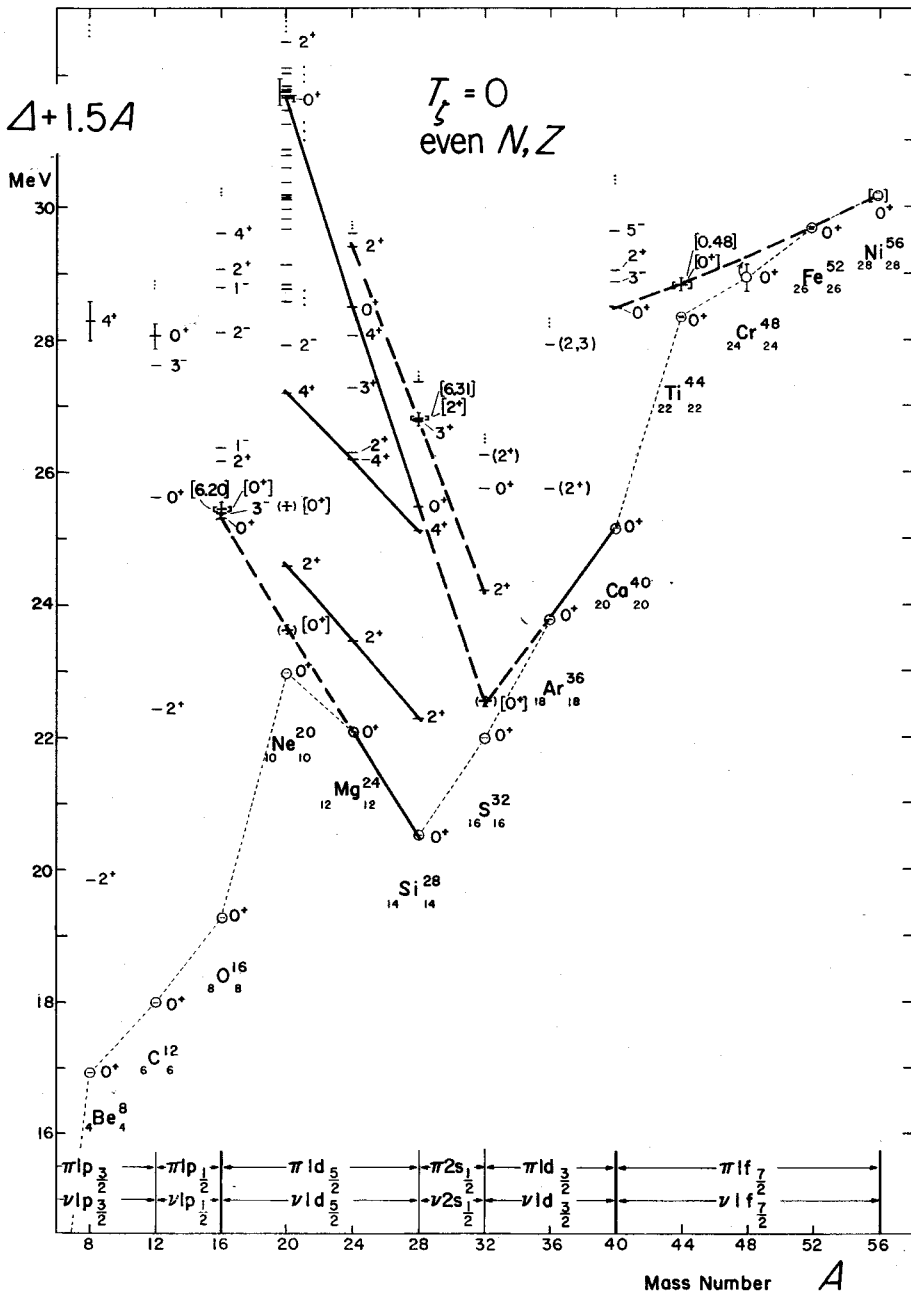


Fig. 1. Nuclidic mass excess  $\Delta$  for ground states (circles) and excited states (dashes), with a linear function  $1.5A$  added, versus the mass number  $A$ . Parentheses indicate doubtful experimental data, brackets indicate expectations if this systematic holds without exceptions. Uncertainties are given separately for ground state mass excesses and excitation energies ( $\leq 30$  keV if not shown). Heavy lines show possible trends (dashed if doubtful)

other reactions. However, it is not yet ruled out by using the same reaction  $F^{19}(d, n)Ne^{20}$ . Therefore there is either a peculiar shift of the  $Ne^{20}$  ground state or an additional  $0^+$  level which would lead the trend to  $O^{16}$  6.05 MeV,  $0^+$ . In any case, a step occurs at  $N, Z = 8$ . Similarly,

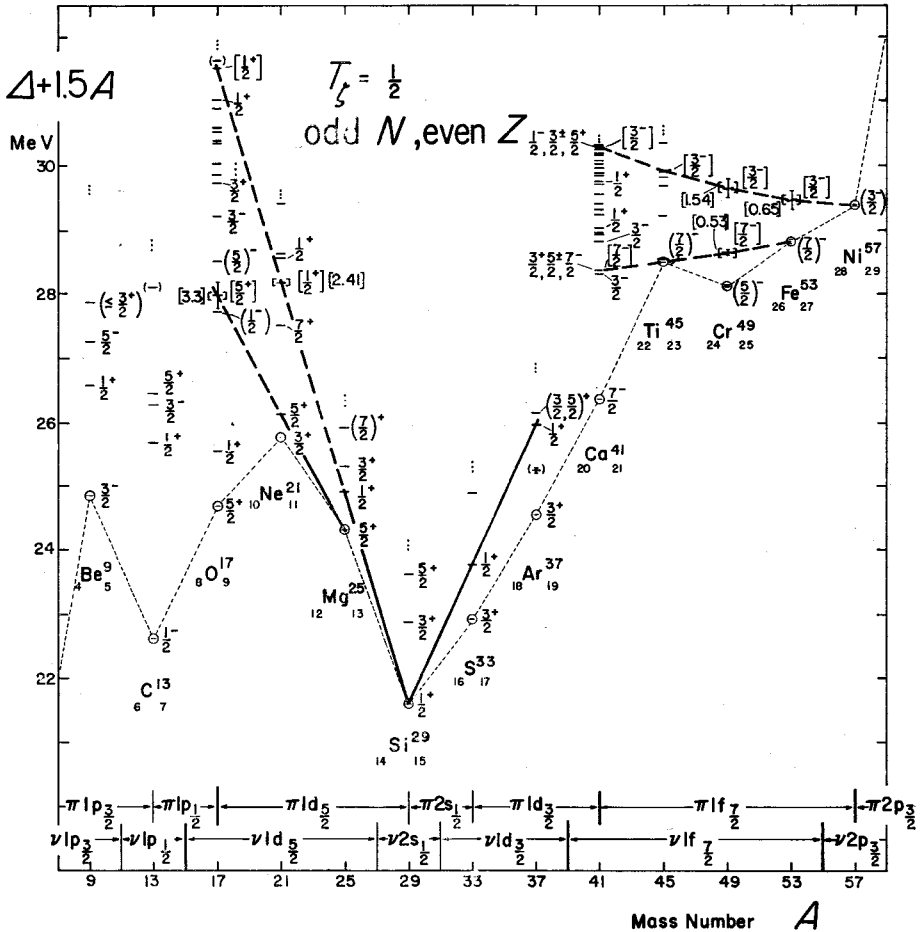


Fig. 2. Odd-neutron nuclei with  $T_z = 1/2$ . For conventions, see Fig. 1

there is a step at  $N, Z = 20$  with  $Ti^{44}$  corresponding to  $Ne^{20}$ . The mass excess of  $Ni^{56}$  is our prediction from systematics of nuclides with  $N = 28$ .

The  $0^+$  trend through *excited* states of  $Si^{28}$  and  $Mg^{24}$  apparently represents configurations with a filled  $2s_{1/2}$  subshell (for protons and neutrons) while the  $1d_{5/2}$  subshell is being occupied. If the tentatively indicated level  $Si^{28}$  6.31 MeV,  $2^+$  exists, the energy required for exciting these  $2^+$  states from the  $0^+$  states below them would be independent of the additional  $2s_{1/2}$  alpha-particle. In  $S^{32}$ , the ground state is shifted

down by 0.41 MeV if there is no additional  $0^+$  level. Earlier reports of a level at 0.5 and 0.43 MeV respectively have been mainly ruled out except for a slight asymmetry in the  $P^{31}(d, n)S^{32}$  ground state group. From the position  $S^{32}$  0.41 MeV a straight trend leads over  $Ar^{36}$  to  $Ca^{40}$ , representing the occupation of the  $1d_{3/2}$  subshell.

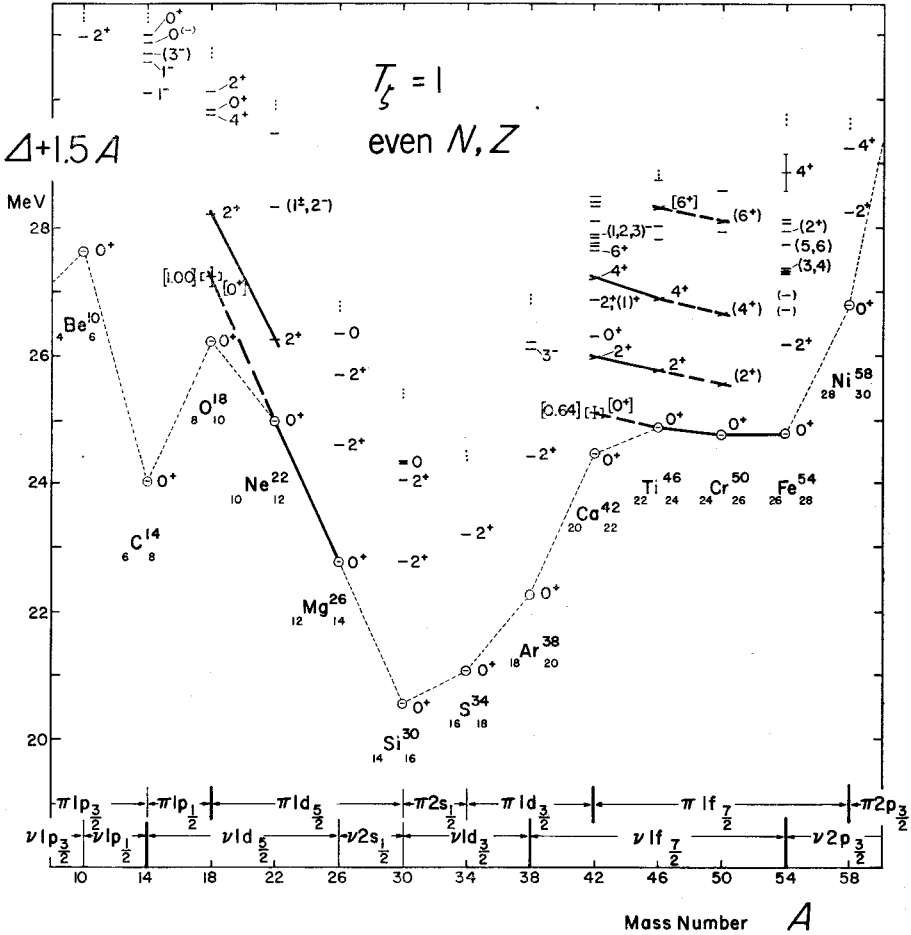


Fig. 3. Even nuclei with  $T_z = 1$ . For conventions, see Fig. 1

Fig. 2 gives these nuclei with a neutron attached. Here the filling of the  $1d_{3/2}$  subshell (while a  $2s_{1/2}$  neutron remains single) is an established straight trend.

A trend through the lowest  $Si^{29}$  and  $Mg^{25} 1/2^+$  states leads in  $O^{17}$  to a level 6.0 MeV above the first  $1/2^+$  level, apparently reproducing the step noted earlier in  $O^{16}$ . The  $5/2^+$  states lead to 3.3 MeV above ground state in  $O^{17}$  where no level is known. In the corresponding  $Ca^{41}$  there is a level at the right position for which  $7/2^-$  is possible.

Fig. 3 shows the nuclides with two neutrons more than the self-conjugate ones. The  $\text{Ca}^{42}$  ground state is shifted down by 0.64 MeV from a smooth trend which is suggested by the sequence of the ground states of  $\text{Fe}^{54}$ ,  $\text{Cr}^{50}$ , and  $\text{Ti}^{46}$  as well as by the  $2^+$  and  $4^+$  trends indicated. An interesting open question is whether the known  $6^+$  level in  $\text{Ca}^{42}$  belongs to the band on the known 1.84 MeV,  $0^+$  level or destroys the pattern suggested. The  $2^+$  and  $4^+$  levels of  $\text{Fe}^{54}$  apparently do not belong to these trends because of the neutron shell closure  $N = 28$ .

Since the  $1d_{5/2}$  subshell is shorter than the  $1f_{7/2}$  shell, we cannot see anything there, except that the downward shift of the  $\text{Ca}^{42}$  ground state has its analogue in  $\text{O}^{18}$ . The level schemes are still too incomplete for locating the states with two neutrons occupying the  $2s_{1/2}$  subshell.

### Conclusions

1. There is a perfect analogy between magic numbers 8 and 20.

2. Smooth trends are likely to exist within subshells which may be destroyed, however, by configuration mixing or other unknown effects immediately after magic numbers 8 and 20.

The almost linear trends mean that about the same energy is liberated when the constituents of an alpha-particle are added repeatedly to a nucleus. The Coulomb repulsion apparently is compensated by the nuclear interactions with particles already in the same subshell.

3. A step exists at these magic numbers in the mass excess (and total nuclear binding energy) surface.

4. If additional levels should turn out to exist and complete these trends, the ground states of  $\text{O}^{17}$  and  $\text{Ca}^{41}$ ,  $\text{O}^{18}$  and  $\text{Ca}^{42}$ , as well as  $\text{Ne}^{20}$  and  $\text{Ti}^{44}$  may have to be considered as cluster configurations of  $\text{O}^{16}$  and  $\text{Ca}^{40}$  in their ground states plus one neutron, two neutrons, and an alpha-particle respectively. The additional states may then contain  $\text{O}^{16}$  and  $\text{Ca}^{40}$  respectively in their first excited state. This may be the reason that they are hard to populate — if they exist at all.

### References

- <sup>1</sup> D. R. INGLIS, *Revs. Modern Phys.* **25**, 390 (1953).
- <sup>2</sup> F. EVERLING, *Nuclear Phys.* **40**, 670 (1963).
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### Discussion

K. BLEULER: In the case of the  $7/2$  levels in light nuclei we have also realized that excited states exhibit a rather continuous behavior as a function of the mass number whereas the position of the corresponding ground states is rather irregular. I was very much interested that you found more examples of this kind.

F. EVERLING: We have tried to extend the curves as far as possible, but when going from the  $1 f_{7/2}$  shell to lighter nuclei, of course, it does not work so accurately because of the residual interaction. The fact that there are sometimes anomalous ground states is a difficulty in the work on mass formulas. One of these, the ground state of  $\text{Cr}^{49}$ , was shown in Fig. 2, and I predicted the unknown  $7/2^-$  single particle level by interpolation. In the odd-proton nucleus  $\text{Mn}^{51}$ , I did the same and learned recently from ARNELL and STERNER that they just found a level at this position for which the spin  $7/2^-$  is not yet confirmed, however. This state could also be predicted from the parabola of nuclei with 26 neutrons which leads to the same value. For states immediately after magic numbers, such trends are probably destroyed by configuration mixing.